Finding n^{th} Roots of Complex Numbers

MATH MONKS

If $z = r(\cos\theta + i\sin\theta)$, then the nth root of z is

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)\right]$$

where $k = 0, 1, 2, \ldots, n - 1$

1. Determine the cube roots of $z = 27(\cos\pi + i\sin\pi)$

2. Find the fourth roots of $z = 5(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

3. Determine the cube roots of $z = -1$

4. Find the eighth roots of unity

5. Find the cube roots of $-2 - 2\sqrt{3}i$

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Answers

1. Determine the cube roots of $z = 27(\cos\pi + i\sin\pi)$

$3(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$, $3(\cos\pi + i\sin\pi)$, $3(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3})$

2. Find the fourth roots of $z = 5(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

$\sqrt[4]{5}(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12})$, $\sqrt[4]{5}(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12})$, $\sqrt[4]{5}(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12})$, $\sqrt[4]{5}(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12})$

3. Determine the cube roots of $z = -1$

$\frac{1}{2} + \frac{\sqrt{3}}{2}i$, -1, $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

4. Find the eighth roots of unity

1, $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$, i, $-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$, -1, $-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$, $-i$, $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

5. Find the cube roots of $-2 - 2\sqrt{3}i$

$\sqrt[3]{4}(\cos\frac{4\pi}{9} + i\sin\frac{4\pi}{9})$, $\sqrt[3]{4}(\cos\frac{10\pi}{9} + i\sin\frac{10\pi}{9})$, $\sqrt[3]{4}(\cos\frac{16\pi}{9} + i\sin\frac{16\pi}{9})$